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A number of studies (cf. [1, 2]) have examined the flow of an incompressible viscous liquid around a rotating sphere, together with magnetohydrodynamic flow around a slowly rotating sphere [3, 4]. In [5, 6] turbulent flows were considered, arising in a conductive incompressible liquid under the influence of the electromagnetic field created by a variable dipole located within a nonconducting sphere. In [5] the dipole was located in the center of the sphere, while in [6] it was shifted away from the center, leading to motion of the sphere relative to the liquid at rest at infinity. The present study will consider the problem of slow flow of a conductive incompressible viscous liquid around a sphere containing a rotating magnetic dipole. The liquid occupies all of an infinite space outside the sphere of radius $\alpha$, as in $[5,6]$. The problem will be solved for the case of small hydrodynamic and magnetic Reynolds numbers. The solution contains two terms of the Stokes expansion.

Let the local current distribution located in the center of the sphere create in a coordinate system fixed to the sphere the rotating magnetic moment

$$
\mathbf{m}=m_{0}\left(\mathbf{e}_{y^{\prime}}+i \mathbf{e}_{x^{\prime}}\right) \exp (i \lambda t)
$$

We assume that the frequency satisfies the quasistationary condition, i.e., $\lambda \alpha / c \ll 1$, where $c$ is the speed of light. Assume that in the laboratory coordinate system the sphere rotates with an angular velocity $\Omega e_{z}$, and that $\Omega \alpha / c \ll 1$, so that the magnetic moment rotates with a frequency $\omega=\lambda-\Omega$ :

$$
\begin{equation*}
\mathbf{m}=m_{0}\left(\mathbf{e}_{y}+i \mathbf{e}_{x}\right) \exp (i \omega t) \tag{1}
\end{equation*}
$$

The vector potential $\mathbf{A}_{m}$ of the electromagnetic field of the dipole of Eq. (1) has the following projections in a spherical coordinate system in free space:

$$
\begin{equation*}
A_{m r} \equiv 0, \quad A_{m \theta}=\frac{m_{0}}{r^{2}} \mathrm{e}^{i \omega t-i \alpha}, \quad A_{m \alpha}=-i \frac{m_{0} \cos \theta}{r^{2}} \mathrm{e}^{i \omega t-i \alpha} \tag{2}
\end{equation*}
$$

where $r, \theta, \alpha$ are the coordinates of the spherical system.
Let the sphere be surrounded by a liquid with conductivity $\sigma$ and magnetic and dielectric permittivities equal to unity. We will find the electromagnetic field distribution. Let $A$ be the electromagnetic field vector potential. Then the current density in the liquid

$$
\mathbf{j}=\sigma(\mathbf{E}+\mathbf{v} \times \mathbf{B} / c)=\sigma(-i \omega \mathbf{A}+\mathbf{v} \times \operatorname{rot} \mathbf{A}) / c
$$

We make the following assumptions:

$$
\begin{equation*}
\operatorname{Re}=a v_{0} / v \ll 1, \operatorname{Re}_{m}=4 \pi \sigma v_{0} a / c^{2} \ll 1,|\mathbf{E}| \gg|\mathbf{v} \times \mathbf{B}| / c \tag{3}
\end{equation*}
$$

where $\alpha$ is the sphere radius; $v_{0}$ is the characteristic flow velocity; $v$ is the kinematic viscosity of the liquid; Re and Rem are the hydrodynamic and magnetic Reynolds numbers. To satisfy the third assumption of Eq. (3) it is sufficient that $\omega a \gg v_{0}, \omega \delta>v_{0}$, where $\delta=$ $c / \sqrt{2 \pi \sigma \omega}$ is the skin layer thickness in the liquid.

We denote the regions within and outside the sphere by I and II, respectively. The $A$ vector potential is defined by equations and boundary conditions

$$
\begin{align*}
& \mathbf{A}_{\mathbf{1}}=\mathbf{A}_{m}+\mathbf{G}, \Delta \mathbf{G}=0, \partial \mathbf{A}_{2} / \partial t=\left(c^{2} / 4 \pi \sigma\right) \Delta \mathbf{A}_{2},  \tag{4}\\
& \mathbf{A}_{\mathbf{1}}=\left.\mathbf{A}_{\mathbf{2}}\right|_{r=a}, \frac{\partial A_{1 \alpha}}{\partial r}=\left.\frac{A \partial_{\alpha 2}}{\partial r}\right|_{r=a}, \frac{\partial A_{10}}{\partial r}=\left.\frac{\partial A_{2 \theta}}{\partial r}\right|_{r=a}
\end{align*}
$$

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Fig. 1


Fig. 2


Fig. 3

where $\boldsymbol{A}_{1}, \boldsymbol{A}_{2}$ are the vector potentials in regions $I$ and $I I ; \boldsymbol{A}_{m}$ and $G$ are the vector potential in $I$ created respectively by magnetic dipole (1) and the currents in the liquid; $A_{m}$ is given by Eq. (2). The solution of Eq. (4) has the form

$$
\begin{gathered}
\mathbf{G}=\frac{i m_{0}}{a^{3}}\left(1-\frac{3 H_{3 / 2}^{(2)}\left(\frac{1-i}{\delta} a\right)}{\frac{1-i}{\delta} a / I_{5 / 2}^{(2)}\left(\frac{1-i}{\delta} a\right)}\right)\left(i r \mathbf{e}_{\theta}+r \cos \theta \mathbf{e}_{\alpha}\right) \mathrm{e}^{i \omega t-i \alpha}, \\
\mathbf{A}_{2}=-\frac{3 i m_{0}}{a^{2}}\left(\sqrt{\left.\frac{1-i}{\delta} a L_{5 / 2}^{(2)} \frac{1 \cdots i}{\delta} a\right) H_{3 / 2}^{(2)}\left(\frac{1-i}{\delta} r\right) / 1 \frac{1-1}{\delta} r\left(i \mathbf{e}_{0}+\cos \theta \mathbf{e}_{\alpha}\right) \mathrm{e}^{i \omega t-i \alpha},}\right.
\end{gathered}
$$

where $\mathrm{H}_{3}^{(2)}, \mathrm{H}_{5}^{(2)}{ }^{(2)}$ are Hankel functions.
Upon a unit liquid volume there acts a force $\mathbf{f}==\mathbf{j} \times \mathbf{B} / c(\mathbf{B}=\operatorname{rot} \mathrm{A})$, having a stationary component and a component oscillating at frequency $2 \omega$. The liquid flow will also have stationary and oscillating components. In order that the maximum of the dimensionless nonstationary velocity be $\ll 1$, it is sufficient that $2 \omega \mathrm{a}^{2} / \nu \gg \max \left(1, a^{2} / \delta^{2}\right)$, which is easily obtained from the linearized nonstationary Navier-Stokes equation. This condition can be rewritten in the form $4 \mathrm{Re} / \mathrm{Re} \mathrm{e}_{\mathrm{m}} \gg \max \left(1, \delta^{2} / a^{2}\right)$. As was noted in [5], for all conductive liquids, including electrolytes and liquid metals, $R e / \mathrm{Re}_{\mathrm{m}} \gg 1$, so that it is sufficient to take $\delta \leqslant a$, in order that the contribution of the nonstationary flow be negligible. We will consider only stationary flows, described by the equations

$$
\begin{equation*}
\mathbf{v}_{\nabla} \mathbf{v}=-\nabla p / \rho+v \Delta \mathbf{v}+\mathbf{f} / \rho, \operatorname{div} \mathbf{v}=0 \tag{5}
\end{equation*}
$$

where $p$ is the pressure and $\rho$ is the liquid density. The force component $f_{\alpha}$ is composed of a stationary part

$$
\begin{equation*}
f_{\alpha}^{+}=f_{0}\left(\left|H_{3 / 2}^{(2)}\left(\frac{1-i}{0} r\right) /\left(r H_{5 / 2}^{(2)}\left(\frac{1-i}{\delta}\right)\right)\right|^{2}\right) \sin \theta, \quad f_{0}=\frac{9 m_{0}^{2}}{4 \pi a^{2}}, \tag{6}
\end{equation*}
$$

where $f_{0}$ is the characteristic value of the force acting on a unit volume of liquid. The expression in the inner parentheses of Eq. (6) is dimensionless, and $r$ and $\delta$ will be dimensionless here and below.

From the symmetry of the problem it is evident that the total electromagnetic force acting on the entire liquid as a whole is equal to zero. The total moment of the electromagnetic forces is not equal to zero, so that we take $\left.\mathbf{v}\right|_{r=t}=-\omega_{0} \sin \theta \mathrm{e}_{\alpha},\left.v\right|_{r \rightarrow \infty} \rightarrow 0$, where $\omega_{0}=\Omega_{a} / \mathrm{v}_{0}$. We rewrite Eq. (5) in dimensionless variables

$$
\begin{equation*}
\operatorname{Rev}_{\nabla} \mathbf{v}=-\nabla p+\Delta \mathbf{v}+T_{\boldsymbol{\varphi}}, \operatorname{div} \mathbf{v}=0 \tag{7}
\end{equation*}
$$

where $\varphi=\left|H_{3 / 2}^{(2)}\left(\frac{1-i}{\delta} r\right) /\left(r H_{5_{2}^{2}}^{(2)}\left(\frac{1-i}{\delta}\right)\right)\right|^{2} \sin \theta \mathbf{e}_{\alpha} ; \quad T=f_{0} a^{2} /\left(\rho v v_{0}\right)$. We will seek a solution of Eq. (7) in the form $\mathbf{v}=\mathbf{v}^{0}+\operatorname{Re} \mathbf{v}^{\mathbf{1}}, \rho=p^{0}+\operatorname{Re} p^{1}$. Considering Eq. (3), for $\mathbf{v}^{0}, p^{0}$ we obtain a system of equations and boundary conditions

$$
-\nabla p^{0}+\Delta \mathbf{v}^{0}+T \varphi=0, \operatorname{div} \mathbf{v}^{0}=0,\left.\mathbf{v}^{0}\right|_{r=1}=-\omega_{0} \sin \theta \mathbf{e}_{\alpha},\left.\mathbf{v}^{0}\right|_{r \rightarrow \infty} \rightarrow 0
$$

The solution of this system has the form

$$
\begin{gather*}
p^{0}=\mathrm{const}, \quad v_{\theta}^{0} \equiv v_{r}^{0} \equiv 0, \quad v_{\alpha}^{0}=R(r) \sin \theta, \quad R(r)=r M(r)-M(1) / r^{2}-\omega_{0} / r^{2},  \tag{8}\\
M(r)=-M_{0}\left(\frac{1}{r^{4}} \mathrm{e}^{2(1-r) / 6}+\frac{2}{r^{4}} \int_{\infty}^{1} \frac{d t}{\delta^{5}} \mathrm{e}^{2(1-r) / 8}\right), \\
M_{0}=\frac{f_{0} a^{2}}{\rho v_{0}} \frac{\delta^{2}}{4\left((1+1.5 \delta)^{2}+\left(1.5 \delta+1.5 \delta^{2}\right)^{2}\right)} .
\end{gather*}
$$

From the solution obtained it is evident that the characteristic velocity is the value $\mathrm{v}_{0}=$ $\left(\mathrm{f}_{\rho} \alpha^{2} /(\rho \nu)\right) \delta^{2} /\left(4\left((1+1.5 \delta)^{2}+\left(1.5 \delta+1.5 \delta^{2}\right)^{2}\right)\right)$. For $\mathbf{v}^{1}$, $p^{1}$ from Eq. (7), considering Eq. (3), we obtain the system $\mathbf{v}^{0} \nabla \mathbf{v}^{10}=-\nabla p^{1}+\Delta \mathbf{v}^{1}, \operatorname{div} \mathbf{v}^{\mathbf{1}}=0,\left.\mathbf{v}^{1}\right|_{r=1}=0,\left.\mathbf{v}^{1}\right|_{r \rightarrow \infty} \rightarrow 0$, the solution of which has the form

$$
\begin{equation*}
v_{r}^{\mathrm{1}}=R_{1}(r) P_{2}(\cos \theta), \quad v_{\theta}^{1}=R_{2}(r) P_{2}^{1}(\cos \theta), \quad v_{\alpha}^{1} \equiv 0, \tag{9}
\end{equation*}
$$



Fig. 5

$$
\begin{gathered}
R_{1}(r)=-\frac{1}{3 r^{2}} \int_{1}^{r} R^{2}(\xi) \xi^{2} d \xi+\frac{1}{5 r^{4}} \int_{1}^{r} R^{2}(\xi) \xi^{4} d \xi+\frac{2 r}{15} \int_{\infty}^{r} R^{2}(\xi) d \xi / \xi+\left(\frac{1}{5 r^{4}}-\frac{1}{3 r^{2}}\right) \int_{\infty}^{2} R^{2}(\xi) d \xi / \xi, \\
R_{2}(r)=-\frac{1}{3} R_{1}(r)-\frac{r}{6} \frac{d R_{1}}{d r}, \quad P_{2}(\cos \theta)=\frac{3 \cos ^{2} \theta-1}{2}, \quad P_{2}^{1}(\cos \theta) \cdots 3 \sin \theta \cos \theta .
\end{gathered}
$$

We will not present the expression for $p^{3}$. The integrals appearing in the expressions for $R(r), R_{1}(r), R_{2}(r)$ can be obtained numerically. We will only estimate these. Let $\delta \mathbb{K} 1$. We expand $R(r), R_{1}(r), R_{2}(r)$ in asymptotic series. We will consider two variants of the problem. Let the sphere be rigidly held and unable to rotate. Then from Eqs. (8), (9) we obtain

$$
\begin{gather*}
R(r)=\left(\frac{1}{r^{2}}-\frac{e^{2(1-r) / \delta}}{r^{3}}\right)+\delta\left(\frac{e^{2(1-r) / \delta}}{r^{4}}-\frac{1}{r^{2}}\right)  \tag{10}\\
+\delta^{2}\left(\frac{5}{2 r^{2}}-\frac{5}{2 r^{6}} \mathrm{e}^{2(1-r) / \delta}\right)+\delta^{3}\left(\frac{15}{2 r^{6}} \mathrm{e}^{2(1-r) / \delta}-\frac{15}{2 r^{2}}\right)+O\left(\delta^{4}\right), \\
R_{1}(r)=(1-2 \delta)\left(-\frac{1}{4 r^{2}}+\frac{1}{2 r^{3}}-\frac{1}{4 r^{4}}\right) \div \delta^{2}\left(-\frac{17}{16 r^{2}}+\frac{3}{r^{3}}-\frac{31}{16 r^{4}}\right) \\
+\delta^{3}\left(\frac{29}{16 r^{2}}-\frac{10}{r^{3}}+\frac{247}{32 r^{4}}+\frac{1}{2 r^{7}} e^{2(1-r) / \delta}-\frac{1}{32 r^{8}} \mathrm{e}^{4(1-r) / 8}\right)+O\left(\delta^{4}\right), \\
R_{2}(r)=(1-2 \delta)\left(\frac{1}{12 r^{3}}-\frac{1}{12 r^{4}}\right)+\delta^{2}\left(\frac{1}{2 r^{3}}-\frac{31}{48 r^{4}}+\frac{1}{6 r^{6}} e^{2(1-r) / \delta}\right. \\
\left.-\frac{1}{48 r^{7}} \mathrm{e}^{4(1-r) / 8}\right)+\delta^{3}\left(-\frac{5}{3 r^{3}}+\frac{247}{96 r^{4}}-\frac{1}{6 r^{6}} \mathrm{e}^{2(1-r) / \delta}-\frac{5}{6 r^{7}} \mathrm{e}^{2(1-r) / \delta}\right. \\
\left.+\frac{3}{32 r^{8}} \mathrm{e}^{4(1-r) / 8}\right)+O\left(\delta^{4}\right) .
\end{gather*}
$$

Since $d R_{1} /\left.d r\right|_{r=1}=R_{1}(1)=0$, the smaller $r-1$, the more terms of the asymptotic series that must be calculated to obtain $R_{1}(r)$ to the required accuracy, so that the expression $R_{2}(r)$ in Eq. (10) loses its meaning for $r-1 \leqslant \delta$. To calculate $R_{1}(r)$ for $r-1 \leqslant \delta$ we expand this function in Eq. (9) in a Taylor series. Let $r=1+\varepsilon, \varepsilon \mathbb{K}$. For $\omega_{0}=0$ we obtain

$$
\begin{gather*}
R_{\mathbf{1}}(1+\varepsilon)=\left(\varepsilon^{2}-\frac{8}{3} \varepsilon^{3}+\frac{16}{3} \varepsilon^{4}-\frac{46}{5} \varepsilon^{5}+\frac{237}{15} \varepsilon^{6}-\frac{64}{3} \varepsilon^{7}\right.  \tag{11}\\
\left.+O\left(\varepsilon^{8}\right)\right)\left(-\frac{1}{4}+\frac{5 \delta}{4}-\frac{89 \delta^{2}}{16}+\frac{1978^{3}}{8}+O\left(\delta^{4}\right)\right) .
\end{gather*}
$$

We will analyze the solution obtained. The electromagnetic forces produce a rotating flow in the liquid about the $z$ axis. The inertial forces developed in such a flow generate a secondary flow, obtained as the second term of the Stokes series. This secondary flow describes the adflux of liquid to the sphere in the polar regions (angle $\theta \leqslant 55^{\circ}$ or $\theta \geqslant$ $125^{\circ}$ ) and the departure of liquid from the sphere in the equatorial region. The solution obtained can be compared to that of flow of a viscous incompressible liquid about a slowly rotating solid sphere. The first term of the Stokes expansion for such a problem is contained in Eq. (7). The following terms of the expansion can be written and it can be shown that the solution of the rotating sphere problem in the form of a Stokes series is equally applicable for all r. For $r-1 \gg \delta$ solution (10) is similar to the solution of the rotat-
ing sphere problem, so that it remains applicable as $r \rightarrow \infty$, with following terms of the expansion $\mathbf{v}^{n}$ giving small additions to $\mathbf{v}^{0}, \mathbf{v}^{1}$ at all $r$.

We will consider another variant of the problem: The sphere is not rigidly held and can rotate. From the currents induced in the liquid there acts on the magnetic dipole inside the sphere a moment

$$
\mathbf{M}_{-}=-\mathbf{e}_{z} \frac{3 m_{0}^{2}}{a^{3}} \delta(1+\delta) /\left(1+3 \delta+\frac{9}{2} \delta^{2}+\frac{9}{2} \delta^{3}+\frac{9}{4} \delta^{4}\right) .
$$

The viscosity of the liquid produces a moment acting on the sphere

$$
\mathbf{M}_{+}=\mathbf{e}_{z} \frac{3 m_{0}^{2}}{a^{3}} \frac{\delta\left(1+\delta\left(\frac{3 \omega_{0}}{2}+\frac{1}{4}\right)+\frac{3}{2} \int_{\infty}^{1} \mathrm{e}^{2(1-t) / \delta} d t / t^{4}\right)}{1+3 \delta+\frac{9}{2} \delta^{2}+\frac{9}{2} \delta^{3}+\frac{9}{4} \delta^{4}}
$$

The net moment acting on the sphere must be equal to zero. From this condition we find that $\omega_{0}=1+2 \int_{\infty}^{1} \mathrm{e}^{2(1-t) / \delta} d t / t^{5}$. For $\delta \ll 1$ from Eqs. (8), (9) we find

$$
\begin{gather*}
R(r)=\mathrm{e}^{2(1-r) / \delta}\left(-\frac{1}{r^{3}}+\frac{\delta}{r^{4}}-\frac{5 \delta^{2}}{2 r^{5}}+\frac{15 \delta^{3}}{2 r^{6}}+O\left(\delta^{4}\right)\right),  \tag{12}\\
R_{1}(r)=\delta^{2}\left(\frac{1}{16 r^{4}}-\frac{1}{16 r^{2}}\right)+\delta^{3}\left(\frac{5}{16 r^{2}}-\frac{9}{32 r^{4}}-\frac{1}{32 r^{8}} \mathrm{e}^{4(1-r) / 8}\right) \\
+\delta^{4}\left(\frac{9}{8 r^{4}}-\frac{21}{16 r^{2}}+\frac{3}{16 r^{9}} \mathrm{e}^{4(1-r) / \delta}\right)+O\left(\delta^{5}\right), \\
R_{2}(r)=\delta^{2}\left(\frac{1}{48 r^{4}}-\frac{1}{48 r^{7}} \mathrm{e}^{4(1-r) / \delta}\right)+\delta^{3}\left(\frac{3}{32 r^{8}} \mathrm{e}^{4(1-r) / \delta}-\frac{3}{32 r^{4}}\right) \\
+\delta^{4}\left(\frac{3}{8 r^{4}}-\frac{3}{8 r^{9}} \mathrm{e}^{4(1-r) / \delta}\right)+O\left(\delta^{5}\right) .
\end{gather*}
$$

In analogy to Eq. (11), we find for $\mathrm{r}=1+\varepsilon, \varepsilon \ll 1$

$$
\begin{gathered}
R_{1}(1+\varepsilon)=\varepsilon^{2 \xi}+\varepsilon^{3}\left(R^{2}(1)-8 \xi\right) / 3+\varepsilon^{4}\left(-5 R^{2}(1)+R(1) d R /\left.d r\right|_{r=1}+32 \xi\right) / 6+O\left(\varepsilon^{5}\right), \\
\xi=-\frac{\delta}{4}+\frac{15}{16} \delta^{2}-\frac{27}{8} \delta^{3}+\frac{403}{32} \delta^{4}+O\left(\delta^{5}\right) .
\end{gathered}
$$

We will compare Eqs. (12) and (10). In Eq. (10) the function $R(r)$ contains a term $1 / \mathrm{r}^{2}$, which decreases slowly as $r \rightarrow \infty$, and for large $r$ the flow is similar to that about a rotating sphere [7], while in Eq. (12) the function $R(r)$ decreases exponentially with increase in $r$, the flow is concentrated in a skin layer around the sphere, and differs significantly from the flow of [7]. Just as $\mathbf{v}^{0}, \mathbf{v}^{1}$ were obtained, the following terms $\mathbf{v}^{n}$ of the Stokes series can be derived. Since for $n \geqslant\left. 1 \mathbf{v}^{n}\right|_{r=1}=0$ and $\mathbf{v}_{\sigma}^{0} \sim \mathcal{e}^{\mathrm{e}^{2(1-r) / 8} \text {, all following terms of }}$ the series $\mathbf{v}^{n}$ will decrease as $\mathrm{r} \rightarrow \infty$ no slower than $\mathbf{v}^{0}$ or $\mathbf{v}^{1}$, so that the solution obtained will be applicable for large $r$ also.

To illustrate the flow in the meridional plane we introduce the flow function $\psi(r, \theta)$. We haviv ${ }^{0} y_{r}=\left(1 /\left(r^{2} \sin \theta\right)\right) \partial \psi / \partial \theta, v_{\theta}=(-1 /(r \sin \theta) \partial \psi / \partial r$. We choose $\delta=0.01$. For the first variant ( $\omega_{0}=0$ ) graphs of the functions $R(r),-40 R_{1}(r)$, and $100 R_{2}(r)$ are shown in Fig. 1 (curves $1-3$, respectively). Figure 2 shows isolines $100 \psi(r, \theta)$ in the upper hemisphere ( $\alpha=$ const). The flow in the lower hemisphere is symmetric about the axis $\theta=\pi / 2$. For the second variant ( $\omega_{0} \neq 0$ ) curves of $R(r)$ are shown in Fig. 3, while the functions $-10^{6} R_{1}(r)$ and $10^{6} \mathrm{R}_{2}(\mathrm{r})$ are presented in Fig. 4 (curves 1, 2 respectively). In Fig. 5 the isolines $10^{6} \psi(r, \theta)$ in the upper hemisphere ( $\alpha=$ const) are shown.

In this problem, as in [6], the condition $R e<1$ at $\delta \ll 1$ is equivalent to the condition of smallness of the magnetic field $B_{0}=m_{0} / a^{3}$ :

$$
\frac{9}{16 \pi} \frac{B_{0}^{2} a^{2}}{9 v^{2}}\left(\frac{s}{a}\right)^{2} \ll 1
$$

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## HIGHLY INDUCTIVE EXPLOSIVE-MAGNETIC GENERATORS WITH HIGH ENERGY

GAIN FACTOR
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Spiral explosive-magnetic generators (EMG) are sources of powerful electromagnetic energy pulses [1-3]. One of the most important characteristics governing the practical realization of the spirals is the magnitude of the energy gain factor ( $\mathrm{K}_{\mathrm{E}}$ ). The dimensions of the primary energy source depend directly on the amplifying capabilities of the EMG. Since the specific energy assured by explosive current generators is approximately three orders of magnitude higher than the specific energy of condenser apparatus ordinarily used to power an EMG, the volume of the initial energy source approaches the EMG volume only if the generator energy gain factor reaches $\sim 10^{3}$.

Two possibilities exist for raising the $K_{E}$ of explosive-magnetic units. One is to produce cascade systems that are several EMG connected by using couplers (air transformers) and operating in succession [1-4]. In this case the energy gain factor of the whole system equals the product of the $\mathrm{K}_{\mathrm{E}}$ of each $E M G$ and can reach an arbitrarily high value. However, cascade generators are complex and costly units. Moreover, the presence of couplers considerably increases the size and weight of the system (e.g., the dimensions of an air transformer are commensurate with the dimensions of the EMG itself). Another possibility for obtaining high values of $K_{E}$ is to increase the ratio $\lambda=L_{0} / L_{f}$ (here $L_{0}$ in the initial inductance of the EMG, and $L_{f}$ is the load inductance) by raising $L_{0}$. Construction of the generator is not complicated in practice here. This paper is devoted to namely spirals with high initial inductance.

## 1. Electrical Field during Operation of Highly Inductive Spirals

As is known, electrical fields capable of resulting in the origination of breakdowns and energy reduction in the load are developed in the volume of generators because of the high rate of magnetic field growth with rapid compression of the magnetic flux. In the limit case, the maximal stress in spiral generators tends to the quantity LdI/dt $\simeq I d L / d t=$ ( $\Phi / \mathrm{L}) \mathrm{dL} / \mathrm{dt}$, where L is the inductance, $I$ is the current, and $\Phi$ is the magnetic flux. Especially high voltages are developed in highly inductive spirals since they are powered by a high magnetic flux (for a given flux in the load, the magnitude of the initial flux $\Phi_{0}$ should be the higher, the greater the ratio $L_{o} / L_{f}$ ). Depending on the initial energy, the law of inductance variation, and the size of the system, voltages in an EMG can reach tens

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